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ANALOGY BETWEEN HEAT AND MASS TRANSFER BY NATURAL CONVECTION FROM AIR TO HORIZONTAL TUBES

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Abstract — Experiments were carried out on natural convection heat and mass transfer from air to horizontal tubes in the range 10^3–10^6 for *GrPr* and *GrSc*. Dimensionless equations for both heat transfer (sensible) and mass transfer were suggested. The analogy between heat and mass transfer and the effect of mass transfer on the mechanism of sensible heat transfer were studied.

NOMENCLATURE

- d, diameter;
- A, surface area of tube;
- t_1, inlet temperature of the ethylene glycol;
- t_2, outlet temperature of the ethylene glycol;
- t_0, bulk temperature;
- t_s, average surface temperature of tube;
- C_0, concentration in bulk;
- C_s, concentration at tube surface;
- m, flow rate of the ethylene glycol;
- w, rate of condensed water or sublimed snow;
- h_h, heat transfer coefficient;
- h_m, mass transfer coefficient;
- q_{tot}, total heat transfer;
- q_{sen}, sensible heat transfer;
- q_{lat}, latent heat transfer;
- q_{rad}, heat transfer by radiation;
- L, latent heat;
- k, thermal conductivity;
- D, diffusion coefficient;
- g, gravity constant;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- ρ, density;
- $\Delta\rho$, density difference between bulk and tube surface due to the combined effects of partial pressure and temperature;
- C_p, specific heat of moist air;
- C_{p_e}, specific heat of the ethylene glycol;
- Nu, Nusselt number $(h_h d/k)$;
- Sh, Sherwood number $(h_m d/D)$;
- Gr, Grashof number $(g\Delta\rho d^3/\nu^2\rho)$;
- Pr, Prandtl number $(C_p\mu/k)$;
- Sc, Schmidt number (ν/D);
- Le, Lewis number $(h_h/h_m C_p)$.

INTRODUCTION

HEAT and mass transfer by natural convection from moist air to horizontal tubes is used in many practical applications in refrigeration and air conditioning. The available data for the combined mechanisms, as far as the author knows, are limited. The only data available are on either pure heat transfer by natural convection or pure mass transfer by natural convection. The present research work was carried out to fill this gap.

SUMMARY OF PREVIOUS WORK

Furber[1] found that heat and mass transfer by forced convection from horizontal flat plates could be represented by the following equations:

$$Nu = 0.033\,Re^{0.8}, \tag{1}$$

$$Sh = 0.034\,Re^{0.8}. \tag{2}$$

Yavnel[2] studied heat and mass transfer for air flowing over a disc at snow formation. The size of the disc was rectangular (400 × 250 mm) the temperature of the air was 285–271 K, relative humidity was 65–80% and air velocity was 2.8–6.7 m/s. The refrigerant temperature inside the different sections of the disc varied from 268 to 258 K.

The sensible heat transfer at the point of snow formation on the disc surface could be represented by the equation

$$Nu_s = 0.038\,Re^{0.8}. \tag{3}$$

The equation for heat transfer to the disc in the absence of mass transfer was also suggested as

$$Nu = 0.032\,Re^{0.8}. \tag{4}$$

The suggested equation for the mass transfer was

$$Sh = 0.035\,Re^{0.8}. \tag{5}$$

The ratio between heat and mass transfer coefficients within the range of the experiments varied from 0.21 to 0.25, e.g. was the same as the specific heat of the air and hence the Lewis number could be taken as unity, so the analogy between the heat transfer mechanism and mass transfer mechanism at the refrigeration of air might exist.

De Leeuw den Bouter[3] measured the simultaneous heat and mass transfer in the laminar free convection at a vertical plate in the liquid phase. An electrochemical method was used for the mass transfer. The heat transfer was measured by differential thermal analysis. The results confirmed the use of the theoretically derived composite Grashof numbers.

Goldstein, Sparrow and Jones[4] performed experiments on natural convection mass transfer adjacent to horizontal plane surface using the naphthalene sublimation technique. By assuming analogy between heat and mass transfer, they used the mass transfer experiments corresponding to heat transfer at a heated isothermal upward facing plate or at a cooled downward facing plate. Circular, square and 7:1 rectangular plan-forms were employed in the tests. A common correlation for all three plan-forms was attained by using characteristic lengths equal to the ratio of the surface area to the perimeter. The transfer coefficients predicted by laminar boundary layer theory fell well below the experimental data and exhibited a different dependence on the Rayleigh number. The corresponding correlation equations were

$$Sh = 0.59\,(Ra_m)^{1/4}, \quad Ra_m > 200 \tag{6}$$

$$Sh = 0.96\,(Ra_m)^{1/6}, \quad Ra_m < 200. \tag{7}$$

Wragg and Nasiruddin[5] measured the rates of electrochemical mass transfer by free convection under the influence of simultaneous thermal free convection for upward facing horizontal disc electrodes. Results were correlated by the equations

$$Sh = 1.75\,(ScGR_m)^{0.227} \tag{8}$$

($10^7 < ScGR_m < 5 \times 10^9$) and

$$Sh = 0.163\,(ScGR_m)^{0.33} \tag{9}$$

($5 \times 10^9 < ScGR_m < 10^{11}$), where GR_m was a combined Grashof number for concentration and thermal buoyancy effects

$$GR_m = Gr_m + (Sc/Pr)^{1/2}\,Gr_h. \tag{10}$$

Schutz[6] used an electrochemical method to find the mass transfer on spheres and horizontal cylinders by natural convection. All the points $GrSc < 10^9$ were used for the calculation of a regression line with the gradient 0.25, yielding

$$Sh = 0.53\,(GrSc)^{0.25}. \tag{11}$$

Smith and Wragg[7] measured the rates of mass transfer between an electrolyte and vertical arrays of horizontal cylinders using the limiting current electrolyte technique. Results for single cylinders

$$Sh = 0.56\,(ScGr_{m,d})^{0.25}. \tag{12}$$

Berman[8] found that the analogy between heat and mass transfer might exist during the condensation of water vapour from moist air with an error less than 10% under the conditions: ε_g (relative partial pressure of the dry air to the total pressure) $\geqslant 0.85$ and $\varepsilon_g/\pi_g \geqslant 2.3$ (where π_g is the ratio of the difference between the partial pressures of the vapour at the wall and away from it to the total pressure of the moist air).

APPARATUS

Figure 1 shows the principle of the apparatus used. It consists essentially of an experimental tube (1) cooled by ethylene glycol at atmospheric pressure. The ethylene glycol is circulated in the circuit by a pump (2) and enters the tube at a constant temperature by using a thermostat (3). Two mechanical vapour compression refrigeration units (4) and (5) are used for cooling the ethylene glycol. The minimum attained temperature on the surface of the tube is 255 K. The rate of flow of ethylene glycol is adjusted by a valve (6) to give a nearly constant temperature on the surface of the tube. The air inside the experimental tube can be removed from the upper point by using a valve (7).

The discharge rate of ethylene glycol can be measured by a flowmeter (8). The surface temperatures of the tube are measured by nine thermocouples which are located at different places as shown in Fig. 1. The mean surface temperature at the tube may be assumed equal to the average of those measured. The drop in temperature of the ethylene glycol through the tube

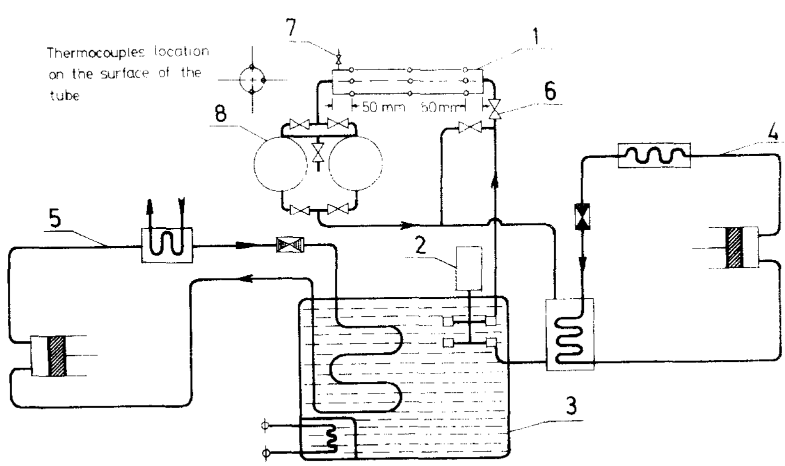

FIG. 1. Diagrammatic sketch of apparatus.

can be measured by a differeniial thermocouple. The dry and wet bulb temperatures of the air are measured by a dry bulb thermometer and a wet bulb thermometer. The condensed water and the sublimed snow on the surface of the tube can be also weighed.

Two tubes of 500 mm length were used. The first tube was 20 mm in diameter and made of brass with a polished bright surface while the second tube was 56.6 mm in diameter and made of steel with a nickel surface. The emissivity of the polished bright brass tube and of the nickel-surfaced steel tube in the dry heat transfer were taken from Perry[9] as 0.023 and 0.045 respectively. The emissivity of the surfaces of both tubes during heat transfer accompanied by condensation of water vapour was taken, also from Perry[9], as 0.95 while their emissivities in heat transfer accompanied by sublimation of water vapour was taken from[10] as 0.85.

PATTERN OF FLOW OF CONVECTION

Three different types of heat transfer by convection from air to the tubes may be obtained according to the condition of the air and the surface temperature of the tube. These types are:

(i) Dry heat transfer without mass transfer may be obtained if the surface temperature of the tube is less than the dry bulb temperature of the air and equal to or higher than the dew point temperature of the air.

(ii) Sensible heat transfer accompanied by latent heat transfer due to condensation of water vapour on the surface of the tube may be obtained if the surface temperature of the tube is less than the dew point temperature of the air but higher than 0°C.

(iii) Sensible heat transfer accompanied by latent heat transfer due to sublimation of water vapour on the surface of the tube may be obtained if the surface temperature of the tube is less than the dew point temperature of the air and lower than 0°C.

The surface temperature of the tube was varied to allow study of the different kinds of heat transfer.

RESULTS AND DISCUSSION

Mass transfer

The weight of the condensed water or the sublimed snow on the surface of the tube enabled the coefficient of mass transfer by convection to be calculated as:

$$w = h_m A(C_0 - C_s). \tag{13}$$

Figure 2 shows the results obtained for the horizontal tubes plotted in the form of $\log Sh$ against log $(GrSc)$. They can be represented by a straight line with a slope equal to 1/4 (indicating the laminar region). The laminar mass transfer ($GrSc$ from 10^3 to 10^6) can be represented, within $\pm$ 14%, by the equation

$$Sh = 0.59\,(GrSc)^{1/4}. \tag{14}$$

Equation (14) gives mass transfer values in the case of combined heat and mass transfer mechanisms 11% higher than the values given by equation (11) suggested by Schutz[6] for the pure mass transfer on horizontal cylinders in the laminar region and 6% higher than that suggested by Smith and Wragg[7].

Heat transfer

The heat lost from the tube was determined by the equation

$$q_{tot} = mC_p(t_1 - t_2). \tag{15}$$

Corrected for radiation q_{rad} and latent heat due to mass transfer q_{lat} the coefficient of heat transfer (sensible) by convection could be calculated as

$$q_{sen} = q_{tot} - q_{rad} - q_{lat} \tag{16}$$

$$= h_h A(t_0 - t_s) \tag{17}$$

where

$$q_{lat} = wL. \tag{18}$$

Figure 3 shows the results obtained of the three different types of heat transfer for the horizontal tubes in the form of log Nu against log $(Gr\,Pr)$. All results can

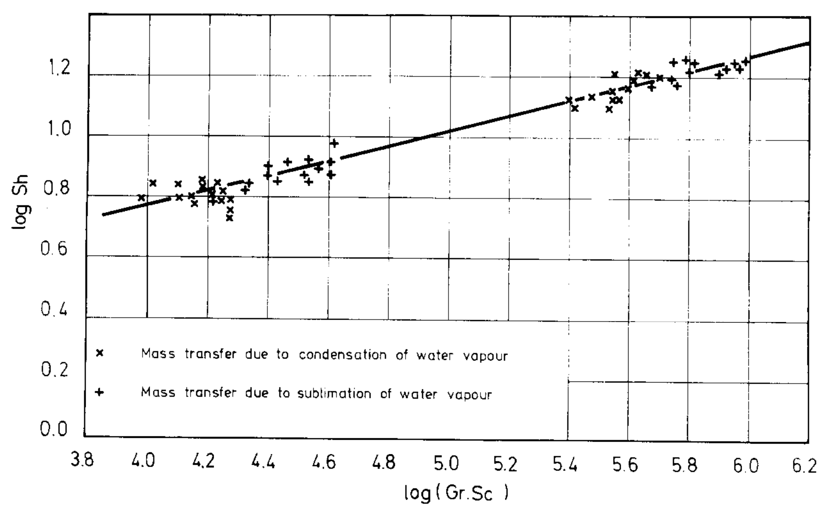

FIG. 2. Variation of $\log Sh$ with $\log(GrSc)$.

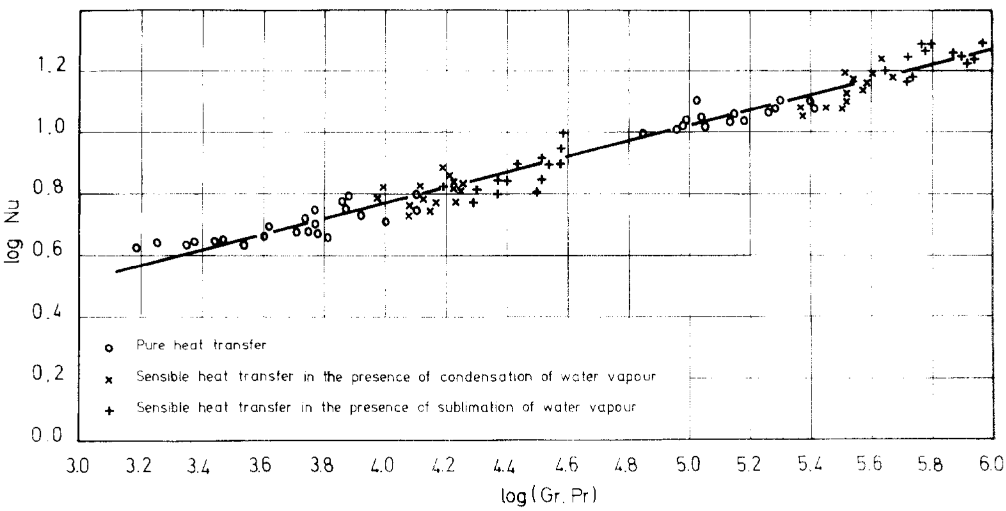

FIG. 3. Variation of log Nu with log $(Gr\,Pr)$.

be represented by one straight line with a slope equal to 1/4 (indicating the laminar region). The laminar sensible heat transfer ($GrPr$ from 2×10^3 to 10^6) can be represented, within $\pm9\%$, by the equation

$$Nu = 0.58\,(GrPr)^{1/4}. \tag{19}$$

Equation (19) gives heat transfer values 5% higher than the values given in the literature for dry heat transfer from the horizontal tubes.

Analogy between heat and mass transfer

As has been seen, the results of sensible heat transfer and mass transfer are nearly the same. This shows that the sensible heat and mass transfer occur simultaneously without any noticeable effect of either of them on the other. Thus, the analogy between sensible heat transfer (pure heat transfer or sensible heat transfer accompanied by mass transfer) and mass transfer in the conditions of the experiment may exist.

It can be seen also that this analogy is verified in forced convection by Furber[1] and Yavnel[2]. The analogy is in good agreement with the present results of combined heat and mass transfer.

The Lewis number, Le, is plotted against the different values of Gr in Fig. 4. Le is found, within $\pm12\%$, to be constant and equal to one.

CONCLUSION

(i) Dimensionless relations have been derived from which heat and mass transfer from air to horizontal tubes in the case of the combined heat and mass transfer can be calculated.

(ii) The presence of mass transfer does not change the mechanism of sensible heat transfer under the conditions used in the experiments and only the total heat transfer will change due to the latent heat transfer.

(iii) The analogy between heat and mass transfer may exist, in the conditions used in the experiments, and the Lewis number can be taken as unity. In view of the analogy, the Sherwood number results of the experiments can be interpreted as if they were Nusselt number results.

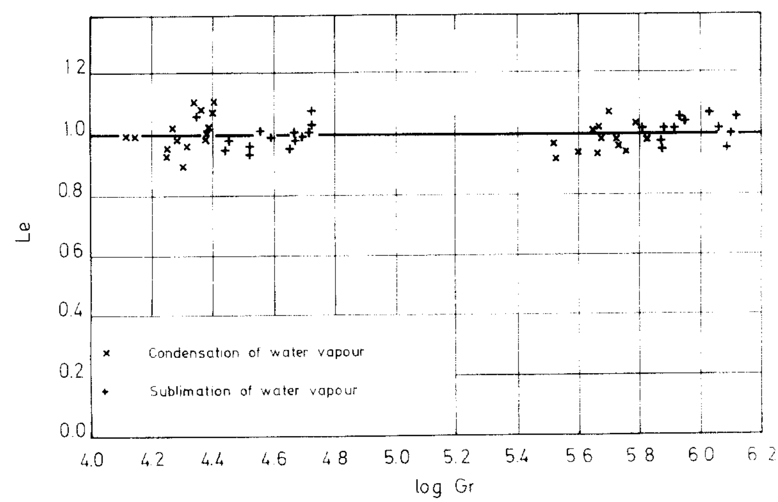

FIG. 4. Variation Le with log Gr.

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ANALOGIE ENTRE LES TRANSFERTS DE CHALEUR ET DE MASSE EN CONVECTION NATURELLE AUTOUR DE TUBES HORIZONTAUX

Résumé—Une étude expérimentale est faite sur la convection naturelle de l'air autour de tubes horizontaux dans le domaine de $GrPr$ et de $GrSc$ compris entre 10^3 et 10^6. Des équations adimensionnelles pour les deux transferts thermique et massique sont proposées. On étudie l'analogie entre ces transferts et l'effet du transfert massique sur le mécanisme du transfert de chaleur sensible.

DIE ANALOGIE ZWISCHEN WÄRME- UND STOFFÜBERGANG BEI FREIER KONVEKTION VON LUFT AN HORIZONTALEN ROHREN

Zusammenfassung—Es wurden Experimente zum Wärme- und Stoffübergang bei freier Konvektion von Luft an horizontalen Rohren im Bereich von 10^3–10^6 für $Gr\,Pr$ und $Gr\,Sc$ durchgeführt. Sowohl für den Wärmeübergang (fühlbare Wärme) als auch für den Stoffübergang wurden dimensionslose Gleichungen vorgeschlagen. Die Analogie zwischen Wärme- und Stoffübergang und der Einfluß des Stoffübergangs auf den Mechanismus der Überstragung fühlbarer Wärme wurden untersucht.

АНАЛОГИЯ МЕЖДУ ТЕПЛО- И МАССОПЕРЕНОСОМ ЕСТЕСТВЕННОЙ КОНВЕКЦИЕЙ ОТ ВОЗДУХА К ГОРИЗОНТАЛЬНЫМ ТРУБАМ

Аннотация — Проведено экспериментальное исследование тепло- и массопереноса естественной конвекцией от воздуха к горизонтальным трубам в диапазоне чисел $GrPr$ и $GrSc$ 10^3–10^6. Предложены безразмерные уравнения для переноса тепла (энтальпии) и массы. Исследовались аналогия между тепло- и массообменом и влияние диффузии на механизм переноса тепла.